Adversarially Robust Coloring for Graph Streams

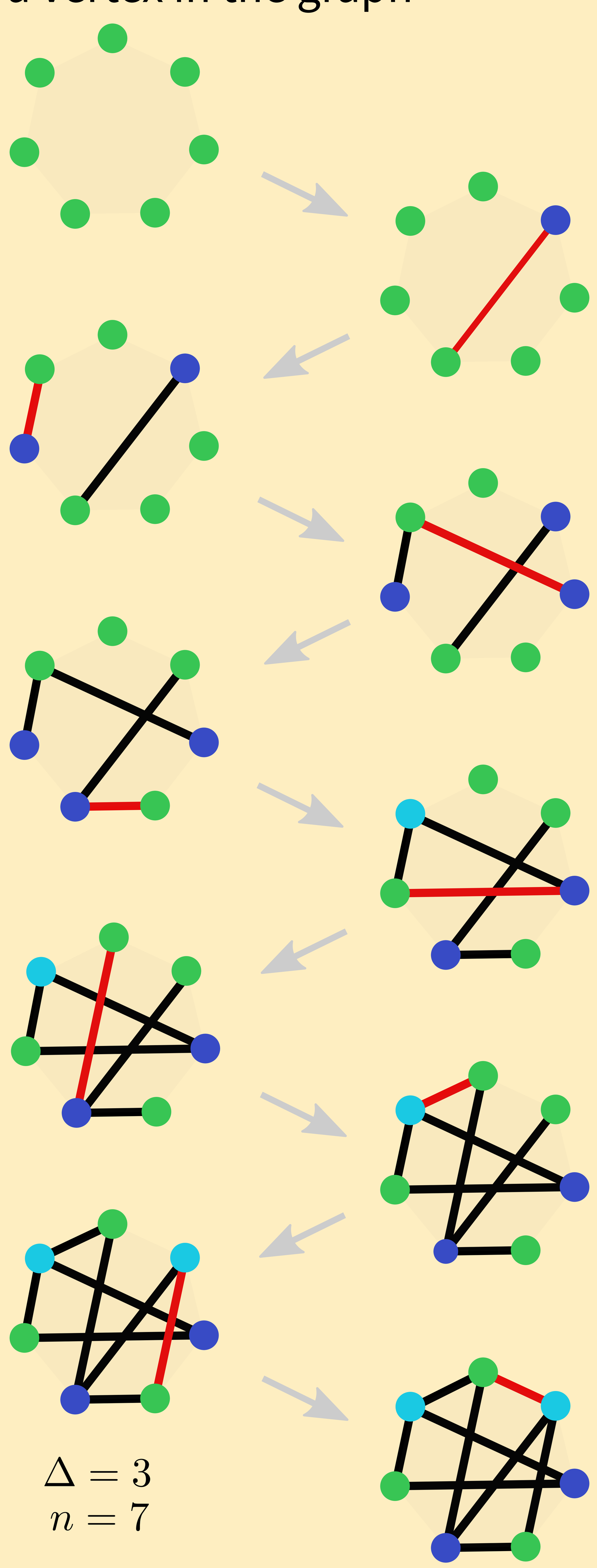
Amit Chakrabarti, Prantar Ghosh, *Manuel Stoeckl* · (Dartmouth College)

Links to paper & poster

Coloring graph streams

Want a data structure using limited space to:

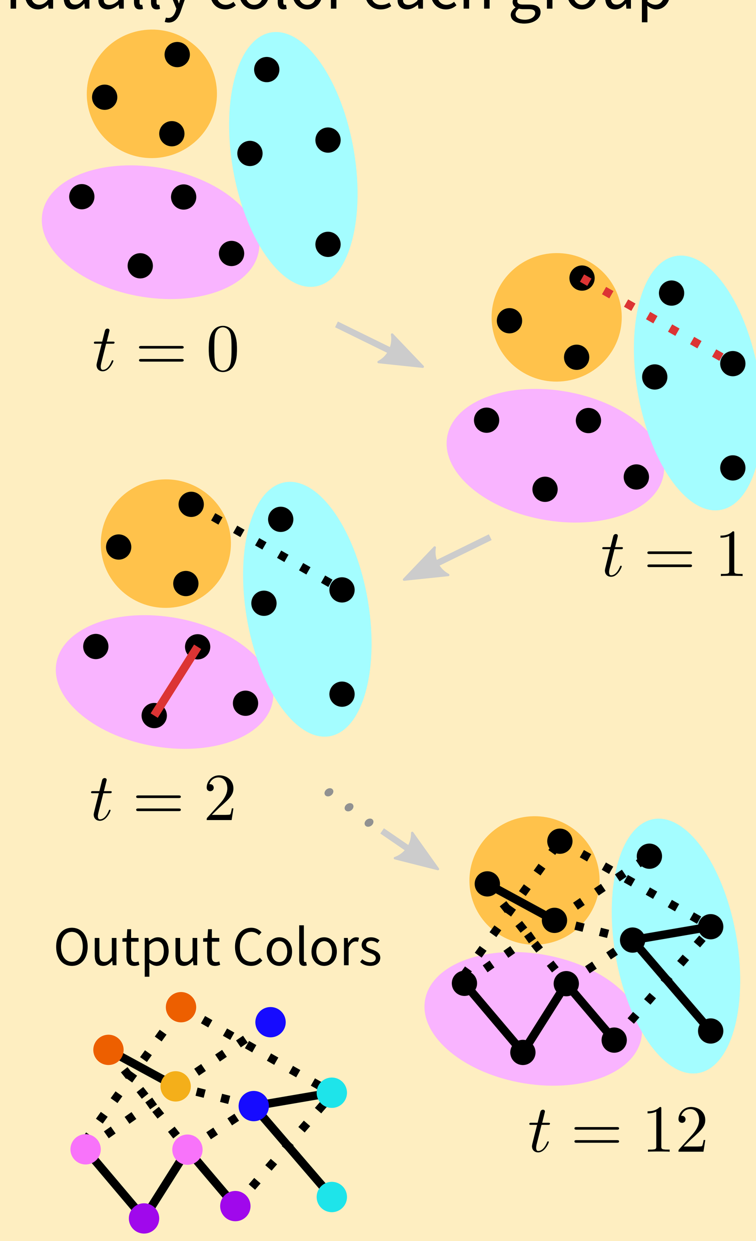
- Receive a sequence of edges of a graph on n vertices
- At all times, assign colors to all vertices with no pair sharing an edge colored alike
- Use a small number of colors in total: $\approx \Delta$, where Δ is the maximum degree of a vertex in the graph



An "oblivious" solution

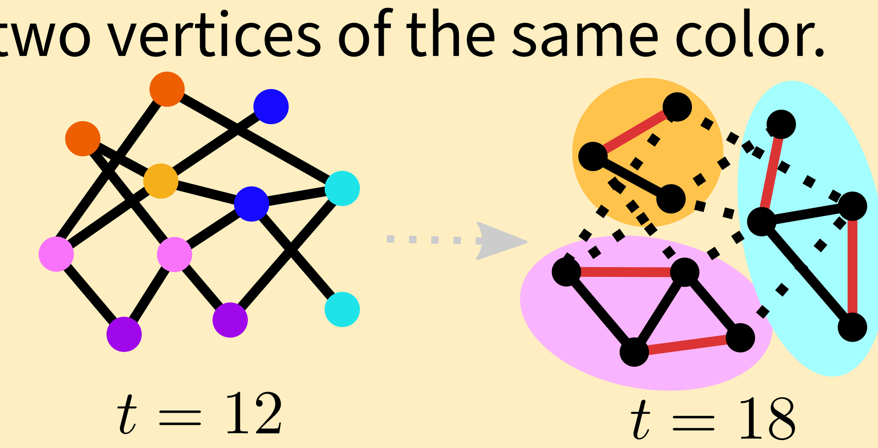
Assuming edges are *independent* of the colors we output, we can solve the problem with $\approx \Delta$ colors and only $\approx n$ bits of space.

- Start: assign each vertex to one of $\Delta / \log n$ groups, at random
- For each edge: if both endpoints are in the same group, record it in the data structure; if not, forget about it
- When asked for a coloring: individually color each group



This data structure fails in the general case:

- Example: always add an edge between two vertices of the same color.

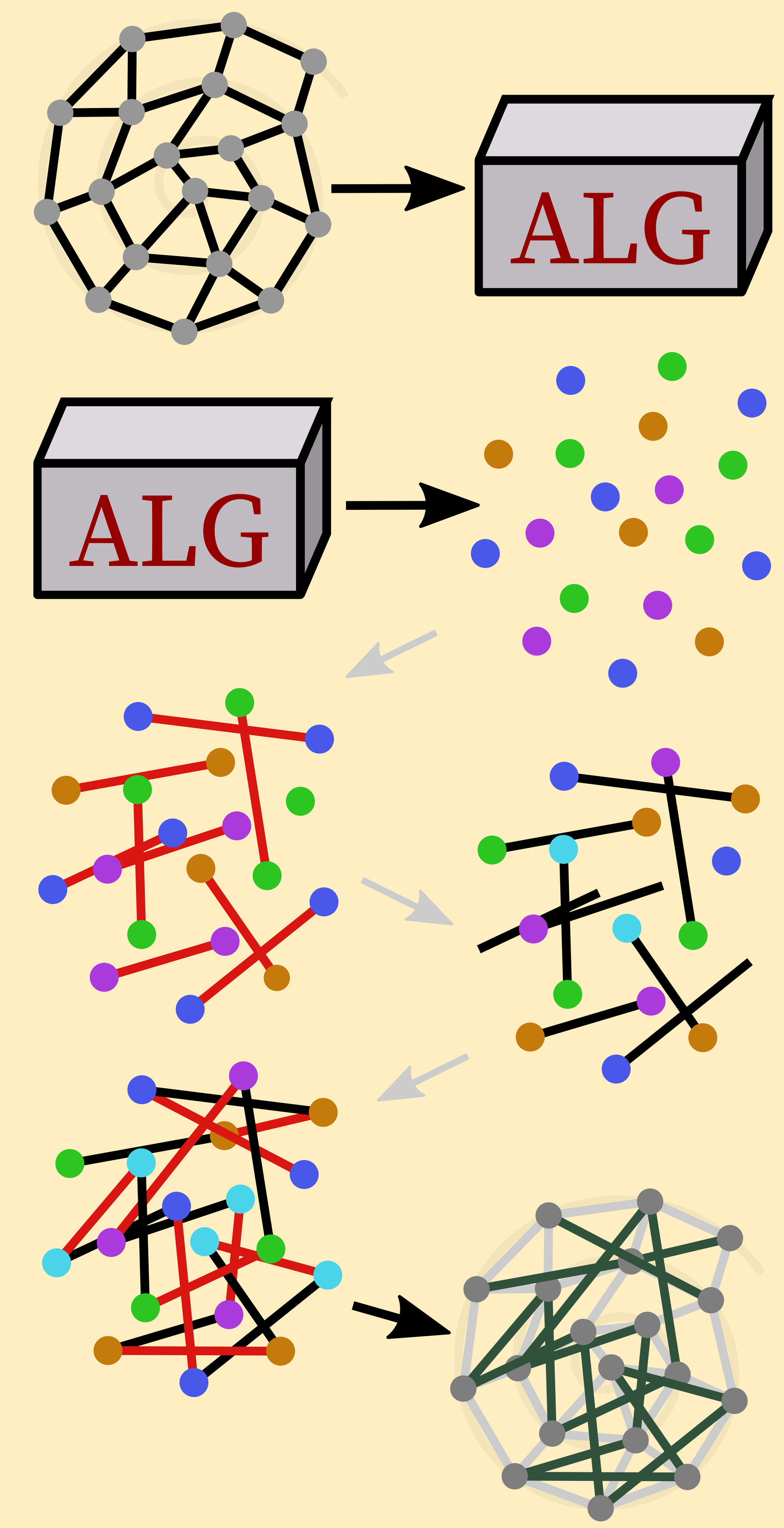


- This tricks the data structure into storing *every* edge and using $\approx n\Delta$ bits of space, instead of $\approx n$

The general/adversarial case is hard

Given a data structure based on a graph, we can recover *many* edges not in the graph.

- Loop:
  - Compute a coloring using data structure
  - Find edges connecting vertices with the same color
  - Add these edges into the data structure



Finding many of the edges *not* in the graph requires about as much space as storing the graph itself.

- Therefore, $\Omega(n\Delta)$ bits of space are needed for $O(\Delta)$ coloring

But we can use Δ^3 colors

This method uses only $\approx n$ bits of space.

- Start: form Δ different random partitions of the vertices into Δ^2 parts, each. Mark one as "active".
- For each edge: store it if both endpoints fall in the same subset of some partition
- Every n new edges, change which partition is "active", and discard the old partition
- When asked for a coloring: color each subset of the active partition with a fresh set of colors